The Race Track Principle in Computational Mathematics

Alkiviadis Akritas

The Race Track Principle is widely used in (Computational) Mathematics but only implicitly. In this talk we will present various different versions of this principle and show how it is used in estimating the round-off errors and in power series approximations of functions.

Vector Topology for Reconstruction, Modelling and Visualization

Chandrajit Bajaj

Volumetric image reconstruction, surface modeling and visualization techniques require that the user infer the global structure from what is frequently an insufficient sampling of data. In this talk I will demonstrate how vector topology computation proves useful for shape reconstruction from imaging, multi-scale modelling and data visualization in diverse scientific applications. I will present a combination of robust numeric and symbolic techniques which help detect structure at all scales, removing from the user the responsibility of extracting information implicit in the data, and presenting the structure explicitly for simulation and analysis. Topology preserving, higher order finite element approximations form a crucial and necessary step in this structure determination, as does the accurate solution of systems of non-linear equations.

Integrated Symbolic-Numerical Approach for Solving Non-Linear Polynomail Systems

Valentine Borisevic

While non-linear polynomial systems (NPS) arise in many areas of modern science and engineering - geometric modeling, modeling physical and chemical objects, robotics, etc. -until now there has been very little use made of so-called "computer algebra" in these applications. In particular, it is connected with the absence of efficient state-of-the-art techniques and algorithms for solving NPS. That was the strong motive for us, people involved in solving various problems of modeling and solving NPS, to start the project dedicated to the development of a new global method for this purpose. Recently this project has been successfully completed. We called the computing technique created "the spectral method" because it is based on the construction of systems of spectral problems for rectangular pencils of matrices [1,2]. This technique is based on the fundamental theory of matrix pencils [3] and connected with the problem of finding solutions for the matrix pencils. This global method for solving NPS with two variables was created as the basic operation of the new technique for calculations. It is designed as an original Toolbox SNAE integrated in MATLAB environment [4]. We have tested the algorithm tailored for the specific problems of flow and multicomponent diffusion calculation in a gas centrifuge for non-uranium isotope separation [5,6].

However, our experience demonstrated that some problems could not be solved by the technique as developed. Now we propose to pursue research goals for a new concept of solving NPS that will overcome the existing difficulties. It will be based on a symbolic-numerical approach for finding all solutions of NPS with more than two variables. One should emphasize that in recent years researchers started to pay attention to this problem as a prospective one for many various applications [7]. Our symbolic-numerical concept will include the following steps. First, the initial system of algebraic equations is transformed in the symbolic arithmetic to a Groebner basis. The algorithm for computing a Groebner basis is known as Buchberger?s Algorithm [8]. With respect to a lexicographic ordering, it is very useful for solving NPS and for the elimination of variables. Also, this symbolic method allows transforming the NPS structure to a form more suitable to solve by other techniques. The problem is that the coefficients in the initial NPS in practice are very often not integer. That is why the calculation technique with the floating point is more reasonable for many practical purposes. Second, the algebraic equations obtained are solved in floating point arithmetic by means of the spectral method. Third, the results obtained are analyzed from the point of view corresponding to the initial problem. The step of reducing the initial system of algebraic equations to one of a Groebner basis is crucial, but it can be done by using the standard computing system MAPLE V. However, the structure of Groebner bases found can be complicated and with a large number of logic branches. Special research is required in order to preprocess efficiently NPS before applying the spectral method.

At the moment, researchers in theoretical physics and quantum chemistry have asked us to give them help in solving their real-life problems that can be presented in the form of algebraic problems. We plan to apply the new algorithm developed on the principles described above to solve these problems. In the framework of this work, bringing together symbolic and numerical algorithms, we would like to incorporate our efforts with those of other researchers that could provide our approach with a solid mathematical foundation. Also, another aim of the project is to deliver implementations of the algorithm developed in a scientific and engineering environment for the solution of systems of polynomial equations for various real problems.

References

[1] Borisevich V.D., Potemkin V.G., Strunkov S.P., and Wood H.G., Direct method for solution of systems of non-linear algebraic equations, submitted to J. Comp. a. Appl. Math. (in review).

[2] Borisevich V.D., Potemkin V.G., Strunkov S.P., and Wood H.G., On global methods for solving systems of non-linear algebraic equations, Comp. a. Math. with Appl., accepted for publication.

[3] Gantmacher F.R. Theory of Matrices, vol. 1 and 2. (Translation by K.A.Hirsch), N.Y., Chelsey Publ. Co., 1959.

[4] Wood H.G., Borisevich V.D., Potemkin V.G., and Strunkov S.P., Spectral method for solving systems of non-linear algebraic equations with two variables (theory and MATLAB toolbox), Rep. No. UVA/537032/MANE99/101, 1998, 61 ps.

[5] Borisevich V.D., Potemkin V.G., Direct method for SNAE solving in modeling of gas flow in centrifuge. - Fourth Workshop Proceedings "Separation Phenomena in Liquids and Gases", August, 19-23, 1994. Tsinghua University, Beijing, P.R. China, 1994, pp.115-127.

[6] Borisevich V.D., Potemkin V.G., Yupatov S.V., Direct method for solving finite difference non-linear equations for multicomponent diffusion in a gas centrifuge. - Fifth Workshop Proceedings "Separation Phenomena in Liquids and Gases", September, 22-26, 1996. Iguassu Falls, Brazil, pp.49-65.

[7] Scientific goals of the initiative (SFB), http://www.can.nl/SN3.

[8] Adams W.W. and Loustaunau P., An Introduction to Groebner Bases, Providence RI: Amer. Math. Soc., 1994.

Theorema - A Status Report

Bruno Buchberger

Co-Authors: Franz Kriftner, Koji Nakagawa, Wolfgang Windsteiger

Theorema is a new software system, based on Mathematica, for mathematical proving, solving, and computing. Mathematical texts of arbitrary size in usual mathematical notation consisting of axioms, definitions, propositions, algorithms etc. can be composed and manipulated in a structured way. Informal text can be intermingled with formal text. Propositions can be proved, formulae can be solved and expressions can be evaluated ('computed') w.r.t. to specific knowledge bases using the various general and special provers, solvers, and evaluators provided by Theorema. The proofs generated automatically by the Theorema provers are presented in the usual style of proofs produced by human mathematicians and contain also informal explanatory text in natural language (English, Japanese; other languages will be added). The Theorema library of provers, solvers, and computers is permanently expanded.

In the talk, we will give an introduction to Theorema by examples that address the user's point of view. In a companion talk, the mathematical and logical methods used in the various Theorema provers, solvers, and computers will be explained.

Theorema is developed, under the direction of B. Buchberger, in the frame of the SFB Project "Scientific Computing" by the Theorema Working Group in interaction with the other working groups of the SFB.

Training of Neural Networks as a Regularization Method

Martin Burger

Co-Author: Heinz W. Engl

We analize network approximation in the framework of approximation and regularization theory. It is shown that training neural networks and similar network approximation techniques are equivalent to least-squares collocation for a corresponding integral equation with mollified data.

Convergence for exact data is examined as well as stability properties with respect to errors in the data. From the stability bounds we obtain, rules for the choice of the number of network elements are derived.

Finally, we give an application to blind signal separation (cooperation with Otmar Scherzer, project F1310) and an outlook to other possible applications in the framework of the SFB F013.

Stochastic Finite and Boundary Element Formulation via Symbolic Computation

Gautam Dasgupta

Randomness in material and boundary geometry introduces second order effects in responses due to any deterministic loading. The straightforward application of the displacement based finite elements and boundary elements with Green's functions for homogeneous media then becomes questionable. Monte Carlo simulation remains to be the only choice when large spatial variability is to be accounted for in practical situations. Real world geotechnical and bioengineering models exhibit such severe randomness. In order to maintain a consistent level of accuracy across the entire Monte carlo sample set stochasticity in the fundamental building blocks of the finite and the boundary element methods are introduced via stochastic shape functions and stochastic Green's functions, respectively. These lengthy algbraic expressions can only be constructed within a computer mathematics environment. The author used the Mathematica software to program the steps demanded by the probabilistic formulation.

The following two-dimensional problems will be demonstrated:

1— high accuracy deterministic cases:

1.a) Deterministic shape functions for convex polygons as rational polynomials

1.b) Exact integration of the energy functional within arbitrary polygons

1.c) Defect-free quadrilateral elements

2— consistent stochastic cases:

2.a) Stochastic shape functions for beams

2.b) Stochastic Green's functions for Helmholtz equation.

Additional Tutorial Session: The programming aspects will be elaborated during an evening tutorial.

Acknowledgement: An NRC-COBASE, USA grant ..... supported the stochastic finite element aspect of the research. An NSF, USA grant partially supported the travel. Encouragement of Professor Franz Ziegler, ATU, Wien, is gratefully acknowledged.

Hybrid Symbolic-Numeric Methods Applied to Definite Integrals and ODEs

Keith O. Geddes

A scientific computation environment such as Maple supports both symbolic and numeric mathematical computation. By exploiting the paradigm of hybrid symbolic-numeric computation, an enhanced level of computational power can be achieved in various problem areas. The strategy is to apply an appropriate combination of symbolic mathematical analysis and numerical computation.

Hybrid symbolic-numeric methods are applied in Maple for the evaluation of definite integrals in the presence of singularities. The hybrid methods will be described, and recent work to enhance the efficiency of the numerical computation phase, by interfacing to NAG numerical routines, will be discussed. For the case of solving ordinary differential equations, we have been studying techniques for generating numerical update formulas for initial-value problems by applying a symbolic calculus of divided differences suggested by W. Kahan.

Computer Algebra Analysis of Constrained Hamiltonian Systems

Vladimir Gerdt

In this talk we consider finite dimensional constrained Hamiltonian systems of polynomial type. Such systems arise in many applications in physics and mechanics and form a special class of differential- algebraic equations. In order to integrate those systems numerically or to investigate their qualitative physical properties one needs to know the complete set of constraints. For application to physics one needs also to separate the constraints into two different classes. For these purposes we apply modern algorithmic methods of commutative algebra based on the use of Gröbner bases. As we show, this makes the classical Dirac method for computation and separation of constraints fully algorithmic. The underlying algorithm implemented in Maple will be presented and some examples from physics and mechanics will be given.

Hierarchical Matrices

Wolfgang Hackbusch

We propose a matrix format which allows to approximate the full system matrices of boundary element matrices or the (full) inverse of a finite element stiffness matrix of an elliptic problem. The technique is closely related to the panel clustering method. The hierarchical structure is given by a cluster tree.

The hierarchical matrix format leads to almost linear cost for the storage as well as for the arithmetical operations of the type matrix-vector multiplication, matrix-matrix multiplication, matrix-matrix addition and matrix inversion, where the latter three operations are performed approximately.

The performance of the matrix arithmetic with almost linear cost opens new and important possibilities, e.g., the complicated iterations involving Schur complements can be replaced by the direct computation of the Schur complement.

The Elasto-Plastic Pendulum: Symbolic and Numerical Computation

Hans Irschik

Co-Authors: Walter Brunner, Johannes Gerstmayr

Dynamics of Elasto-Plastic Structures represent a challenging field of applications for scientific computing. Little work, if any, has been presented so far in the literature concerning elasto-plastic structures with rigid-body degrees-of-freedom. This is due to the fact that, even in the purely elastic case, a strong nonlinear coupling occurs between the flexible and rigid degrees-of-freedom. In the present contribution, we present computational strategies and results for the benchmark problem of an elasto-plastic pendulum. A beam-type pendulum with distributed mass and stiffness is considered within the range of small strains. Symbolic computation is used to derive a non-linear system of ordinary differential-algebraic-equations (DAE) from the field equations. This is performed by means of a Galerkin procedure utilizing the modes of the hinged-hinged elastic background beam. In order to obtain a formulation accurate enough to compute the elasto-plastic strains, a split into quasi-static and dynamic solution parts turns out to be necessary. This split adds an algebraic constraint to the system of ordinary differential equations, requiring vanishing of the shear force of the free end of the pendulum. The DAE system is solved by means of an implicit Runge-Kutta differential algebraic equation solver for index 1 DAEs, developed by our group and written in C++. Efficiency of this procedure is improved by means of automatic differentiation using ADOLC. Plasticity is taken into account by additionally applying eigenstrain loading to the flexural beam. For rate-independent elasto-plastic materials, these eigenstrains correspond to the plastic part of strain. They are computed in an iterative, return-mapping type procedure such that the constitutive equations are satisfied in a sufficiently large number of points within the plastic zone. The pendulum is thus driven by the initial conditions, by its own weight and other external forces, and by the eigenstrains. The computational procedure correspondingly consists of an outer loop, where the DAE system is solved assuming the eigenstrains to be known, and an inner loop, where the eigenstrains are calculated. The computational effort turns out to increase almost linearly with the number of unknown eigenstrains. Flexural motions, bending moments and shear forces of the vibrating pendulum are visualized, together with the distribution of the plastic-zones within the beam.

Proof Methods in Theorema

Tudor Jebelean

**Co-Authors: Bruno Buchberger, Claudio Dupre,
Felix Kossak, Daniela Vasaru-Dupre**

In the Theorema system, various provers, solvers, and computers are offered to the user for proving expressions, finding solutions for expressions, and evaluating expressions in the presence of specified knowledge. The arsenal of provers (solvers, and computers) ranges from general methods (like natural deduction and resolution) that are applicable for a wide range of expressions, but may be less powerful, to specific methods (like Groebner Bases, Cylindrical Algebraic Decomposition, etc.) that are applicable to a limited class of mathematical formulae but are very powerful and efficient.

In the talk, we will explain the basic mathematical and logical ideas behind some of the proving methods used in Theorema, in particular, of those methods that have been developed over the past years within the Theorema Working Group:

* a variant of natural deduction for predicate logic
* the PCS-method for predicate logic
* induction for universally quantified formulae in various inductive domains
* the Groebner Bases method for universally quantified Boolean combinations of equalities over the complex numbers
* functor provers

A particular emphasis will be put on explaining the integration of general and special proof methods into one coherent logical system.

Efficient Algorithms for Computing the Nearest Polynomial with Parametrically Constrained Roots and Factors

Erich Kaltofen

We present efficient, i.e., polynomial-time algorithms in the general area of input-sensitivity analysis: a problem formulation, in our case a polynomial of degree $\leq n$ or an $n \times n$ matrix possibly with floating point coefficients, lacks an expected property because the coefficients are slightly perturbed. A task is to efficiently compute the nearest problem (polynomial or matrix) that has the desired property.

The properties which we consider are:

1. A univariate polynomial of degree $\leq n$ with real or complex coefficients has a root at a given complex point
2. A univariate polynomial of degree $\leq n$ with real coefficients has a real root
3. A univariate polynomial of degree $\leq n$ with complex coefficients has a root on a parameterized curve, e.g., the unit circle
4. A bivariate polynomial of degree $\leq n$ with complex coefficients has a linear complex factor
5. An $n \times n$ square matrix with real entries has a real eigenvalue

For the properties 1 and 2 we give a polynomial-time algorithm when nearness is measured in either the Euclidean distance between the coefficient vectors or, for the real coefficient case, the maximal coefficient-wise change in absolute value (infinity norm). For the properties 3 and 4 we have a polynomial-time solution for the Euclidean distance. For the property 5 we have a polynomial-time solution when measuring nearness in terms of the matrix norms induced by the infinity- or one-norms on the n-dimensional vectors.

We assume that the coefficients, entries, roots, and parameterization are represented in a customary manner, e.g., exact complex rationals and polynomials.

(Joint work with Markus A. Hitz and Lakshman Y. N.)

Scientific Computing Tools

Michael Kuhn

Co-Authors: Michael Schinnerl, Joachim Schöberl

The efficient numerical solution of realistic 3D direct and inverse field field problems, including optimization, requires a wide range of scientific computing tools. Although the numerical solver for the direct field problem has most impact on the overall efficiency, it relies very much on the efficient and well tuned pre– and post–processing of the data.

The preprocessing includes the description and handling of the geometry as well as the generation of a suitable mesh. Since adaptive refinement methods are to be used, additional requirements arise. In particular the full geometry information has to be available throughout the whole simulation process. Applying parallel techniques based on domain decomposition, the preprocessing includes the partitioning of data. In order to make simulation results available, advanced visualization techniques are required. Here, special needs arise from the large amount of data which may involve even several million unknowns.

We present the mesh generator *NETGEN* and the finite element package *FEPP* together with application from structural mechanics, electromagnetics and magneto-mechanics.

Parametrizations of Canal Surfaces

Günter Landsmann

Consider n+1 rational functions $m_1, \ldots, m_n, r \in \mathbf{R}(t)$. The envelope of spheres with centers $(m_1(t), \ldots, m_n(t))$ and radius $r(t)$ is discussed with respect to the existence of a rational parametrization. An algorithm for constructing a rational parametrization is presented. For the case $n = 3$ this gives a method for finding a rational parametrization of canal surfaces.

Analysis of Mechanical Systems via Symbolic and Numerical Computation

Anders Lennartson

Detailed analysis of mechanical systems, i.e. *multibody dynamics*, involve many steps where computer algebra systems, CAS, are useful. Mechanical systems include naturally occurring systems as well as artificial devices. Some examples are planetary motion, biomechanical systems such as flying and walking beings, vehicle dynamics, and mechanical clocks. The objectives of the analysis may be understanding the dynamics, performing optimization, or innovating new products etc. Typical tasks are simulation, locating solutions and bifurcations, and stability computations.

The analysis process can be divided into a symbolic and a numeric domain. System description and application of the principles of mechanics to formulate equations of motion are the main constituents of the symbolic domain. Also, the preparation of equations for the needs of the numerical analyst can be included. This means generating code suitable for efficient numerical work. The complexity of the governing equations often exclude analytical work and numerical methods must be used to compute the tasks mentioned earlier. Accurate stability computations needs differentiation of the original differential equations system to get the Jacobian or the variational equations. CAS are invaluable in this task due to the size of the expressions.

Vector algebra and calculus are the main tools used when describing a system and deriving the equations of motion. It is common to represent vectors by sets of components. Most CAS can be modified to use a more general setting which stresses the vector object rather than the components in some particular reference frame. The components together with information about which reference frame they refer to form a new object which, in essence, is thought of as a vector. By adding functions for specifying relations between reference frames, the operations of vector algebra and calculus can operate seamlessly on vectors defined in different reference frames. Similar methods can be generalized to higher dimensional objects, and one then has an efficient framework for describing complex geometries and their kinematics and dynamics.

Many multibody systems are well described as smooth, but for instance impacting systems undergo discontinuous jumps at certain points in phase space. Others change the behaviour by locking or releasing joints. Such systems can be analyzed by using smooth equations when appropriate and a suitable mapping in phase space during the discontinuity. This type of operations should be supported by good analysis systems.

Quantifier Elimination for Trigonometric Polynomials by Cylindrical Trigonometric Decomposition

Petru Pau

Co-Authors: Josef Schicho

Given a formula Φ in r variables, some of them quantified and/or occurring as arguments in trigonometric functions, we consider in this paper the problem of finding a quantifier-free formula equivalent to Φ.

We present an algorithm that computes first a decomposition of the space so that the polynomials occurring in Φ are sign-invariant over each cell of this decomposition. Then the cells over which Φ is true are collected: their description gives a quantifier-free formula equivalent to Φ. The algorithm is an adaption of a well-known algorithm for the same problem with polynomial functions.

Symbolic Summation: Recent Progress

Peter Paule

The talk gives a status report on results achieved within the SFB project work. The topics range from Karr's summation theory and MacMahon's Partition Analysis to the application of Groebner bases methods for the construction of wavelets.

On Finding Zeros of Complex Polynomials: An Experimental Case Study

Jochen Pfalzgraf

In this contribution we discuss a little project on the combination and interplay of computer algebra, a numerics and a powerful computer graphics system. The objective is the interactive use of these systems to find the zeros of complex univariate polynomials. The idea - as suggested by Professor F.Krueckeberg then at GMD, St.Augustin, Bonn - was to preprocess a given polynomial smbolically (only simple roots), then to visualize the polynomial graphically with the goal to isolate zeros and to exploit this information to find a suitable initial (start) value for a numerical algorithm (Newton) using an interval arithmetic package. The whole process is of experimental nature, of course, and we used the squared absolute value of the polynomials for graphical visualization as smooth surfaces in 3-space. Then the zeros appear as fixed tangent points in the x-y-plane. A lot of scaling, zooming, step size variation (which do not impact the zeros) was necessary to otain suitable optical representations of the polynomials. We examined e.g. polynomial surfaces up to degree 46. We add a remark by Steven Smale about the natural role of the squared absolute value of the polynomials in the whole procedure.

Constructing Multivariate Polynomials with Preassigned Zeros

Shojiro Sakata

In this paper we present an algorithm of finding a Gröbner basis of the ideal composed of multivariate polynomials with preassigned zeros. It is a kind of interpolation problem treated before [1][2]. More precisely, given a finite subset $V := \{\alpha^{(i)} \in \mathbf{F}^k | 1 \leq i \leq n\}$ of the power set $\mathbf{F}^k$ for a field $\mathbf{F} = \mathbf{Q}$ or $\mathbf{R}$ or $\mathbf{C}$, we want to find a Gröbner basis of the ideal $I(V) := \{f \in \mathbf{F}[x] := \mathbf{F}[x_1, \cdots, x_k] | f(\alpha^{(i)}) = 0, 1 \leq i \leq n\}$. The condition $f = \sum_{a \in \mathrm{supp}(f)} f_a x^a \in I(V)$ is equivalent to the system of linear equations $\sum_{a \in \mathrm{supp}(f)} (\alpha^{(i)})^a f_a = 0$, $1 \leq i \leq n$ with respect to the coefficients f_a, $a \in \mathrm{supp}(f)$.

We can show that our problem is reduced to finding a minimal polynomial set of a finite subarray of the array $s = (s_p)$ (over the power set $\mathbf{Z}_0^k$ of the set $\mathbf{Z}_0$ of nonnegative integers) defined by $s_p := \sum_{i=1}^n (\alpha^{(i)})^p (\in \mathbf{F})$, $p \in \mathbf{Z}_0^k$. Cosequently, we can solve the problem efficiently by apply the Berlekamp-Massey-Sakata algorithm.

References

[1] H.M. Moeller and B. Buchberger, The construction of multivariate polynomials with preassigned zeros, in "Computer Algebra: EUROCAM '82 European Computer Algebra Conference, Marseille, France, April 1982," (Ed. J. Calmet), pp.24–31, Lecture Notes in Computer Science, vol.144, 1982.

[2] S. Sakata, N-dimensional Berlekamp-Massey algorithm for multiple arrays and construction of multivariate polynomials with preassigned zeros, in "Applied Algebra, Algebraic Algorithms and Error-Crrecting Codes: AAECC-6, Rome, Italy, July 1988," (Ed. T. Mora), pp.356–376, Lecture Notes in Computer Science, vol.357, Springer Verlag, Berlin, 1989.

Project Report: Proving and Solving Constraints over the Reals

Josef Schicho

The parametrization problem for real algebraic surfaces is important in solving system of equations, but it also has numerous other applications in CAD/CAM. We report on recent progress that has been achieved in this field in the frame of the SFB: A large class of rational surfaces can be reduced to surfaces of a particular shape, called tubular surfaces. For these, we have a parametrization algorithm. Moreover, the proper parametrization problem could be solved completely: if a surface has a proper parametrization, then we are also able to construct one.

Other activities to be reported on are: the complete solution of a problem posed by Millan; the partial solution of a problem posed by Schöberl (see also the joint talk with Schöberl); the study of exact real number arithmetic as a tool which combines symbolic and numeric aspects. In the field of CADs (cylindric algebraic decompositions), we have also obtained new results, which will be reported in Pau's talk.

Quantifier Elimination Problems in the Analysis of Subspace Correction Methods

Josef Schicho, Joachim Schöberl

The solution of large scale numerical field problems requires fast algorithms. Multigrid methods and domain decomposition algorithms are such methods. The analysis of convergence rates involves on one hand side the theory of partial differential equations in Sobolev spaces, and on the other hand side Hilbert space techniques.

It is interesting to observe that many existing proofs in this theory can be given in a formalization of the theory of Hilbert spaces, using only a small number of axioms. Therefore, it seems promising to try to generate such proofs/theorems automatically, using methods from automated theorem proving. However, one has to be careful, because the proximity to the word problem (note that we have to deal with non-commutative algebras!) leads very soon to undecidable questions.

Keeping the dimension a fixed finite number, we can reduce to a quantifier elimination problem over real closed fields. This class of problems can be tracted by existing software. We give examples of these applications.

A Matrix-Analytical Approach for Constructing Wavelets on Intervals

Armin Schoisswohl

Based on the works by Cohen, Daubechies and Vial (ACHA, 1993) and Meyer (Rev.Mat.Iberam., 1991) we discuss constructions of orthonormal wavelet bases on the interval. In our formulation we reduce the constructions to matrix equations which can be solved symbolically and thus avoids any numerical difficulties due to instabilities in previous constructions. This approach can be extended to biorthogonal wavelets in a natural way.

Canonical Form Transformation of Second Order ODE's with Lie Symmetries

Fritz Schwarz

The most difficult part of solving an ordinary differential equation (ode) by Lie's symmetry theory consists of transforming it into a canonical form corresponding to its symmetry type. Although *in principle* this is part of Lie's theory, various intermediate steps are not explicitly given there such that it is not obvious how to apply it to concrete problems. In this article, for all possible symmetry types of a second order ode, theorems are obtained that reduce the transformation to canonical form to solving linear partial differential equations (pde's) or, in some cases, certain Riccati equations. Fundamental new concepts that are applied are the Janet base of a system of linear pde's and its decomposition into completely reducible components, i. e. the analogue to Loewy's decomposition of linear ode's. They allow it to determine *algorithmically* all transformations into canonical form that are Liouvillian over the base field determined by the given ode.

Visualization of Riemann Surfaces

Michael Trott

For over 100 years, Riemann surfaces have been an important concept in mathematics and today enjoy an increasing popularity in modern theoretical physics (in string theories, conformal field theories, etc). Under the direction of many famous mathematicians, including Riemann himself, faithful representations of Riemann surfaces of simple functions in plaster and wood models were created at the turn of the century. Unfortunately, over the last 80 years, the art to produce such models has been forgotten, and with it, the possibility to visualize new functions.

Symbolic manipulation programs, with built-in numerical differential equation solving and visualization capabilities, make it now possible to generate high-quality 3D graphics of virtually every multi-valued function. This talk gives the outline of implementations or the automatic generation of Riemann surfaces of arbitrary algebraic functions and arbitrary compositions of elementary functions (exp, log, power, trigs, and arctrigs) and selected special functions of mathematical physics. Examples of faithful representations of Riemann surfaces of many classes of functions are shown.

Theory of Shells Assisted with MATHTENSOR

Ryszard Walentynski

Theory of shells is a part of theory of elasticity which deals with description and analysis of thin curved elastic bodies called shells. It is based on differential geometry and uses as a tool tensor analysis. The main aim of the theory is to reduce the three dimensional task of the theory of elasticity in the curved space to two dimensional one. It makes possible its practical application to civil and mechanical engineering.

Tensor analysis is a very elegant way of teaching problems of continuum mechanics but is regarded to be very difficult by most of students and engineers. Hand tensor calculations are tedious and prone to error. In the theory of shells some simplifications were done. Some of them were caused by computational difficulties. Therefore the theory is internally inconsistent.

Tensor calculations become more friendly with computer algebra system. The package MathTensor of the Mathematica system has been applied to solve symbolic problems in the theory of shells. The package allows to describe the shell geometrical properties, build and solve its differential equations and so on. The most important achievement was refinement of shells' constitutive equations.

The obtained refined constitutive equations satisfy the last equation of equilibrium. One of the theory inconsistencies was deleded with them. They better describe the shell behaviour. The obtained results with refined and simplified equations in some cases are completly different.

The contribution was supported by Wolfram Research, Inc. with Visiting Scholar Grant 98/99. Most of symbolic and numerical computations were made during the stay in WRI headquarters in Champain, IL in summer 1998.

Matrix Eigenproblem Approach for Positive Dimensional Polynomial System

Wenda Wu

The matrix eigenproblem approach for zero dimensional polynomial system solving in[1] is generalized to positive dimensional case, where a square matrix, not a rectangular one as in [1], is constructed, and by which the zeros of given system can be splitted into several parts. In particular the proposed algorithm is suitable for 1 dimensional system with isolated zeros. For this case a zero dimensional system is computed with those isolated zeros of given system as zeros. Then we can compute them by any numerical method or converting to matrix eigenvector computation. Examples and open problems are presented.

References

[1] H.J.Stetter: Multivariate Polynomial Equations as Matrix Eigenproblem, in : Series in Applicable Analysis (World Scientific) Vol.2, 355-371,1993.

[2] H.M.Moeller, H.J.Stetter: Multivariate polynomial equations with multiple zeros solved by matrix eigenproblems, Numer. Meth. 70, 311-329, 1995.

[3] T.Becker and V.Weispfenning:Groebner Bases—A computational Approach to commutative Algebra, Graduate Texts in Mathematics Vol. 141, Springer Verlag, 1993.

Linear Background Strategies in Nonlinear Dynamics

Franz Ziegler

The paper focuses on structural vibrations and elastic-viscoplastic waves and the accompanying nonlinear P.D.E's and initial - boundary value problems. With the experience of many solved problems at hand, it is recommended to separate a linear operator in the P.D.E. and to consider the nonlinear terms as forcing functions in addition to the given ones. The linear operator, thus, defines a carrier medium which is called the (linear elastic) background. The solution of the unlimited linear case of the background when excited by the given forces is readily available. In the case of vibrations, the solution is split into the quasistatic part, superimposed by the complementary dynamic part commonly in the form of a truncated modal series expansion. The nonlinear source terms in the background require the development of Green's functions according to a single source type. Superposition applies to the increments by stepping the time and discretizing the space. Also that vibrational case needs the decomposition into the quasistatic and the dynamic parts, respectively. In a natural way, as such, the plastic drift is defined.

Waves emitted form these sources require the consideration of causality, e.g., in the Mach-plane. It is shown that even strong plastic shock waves are formed by superpostion of elastic body waves propagating in the background.

Updating in space and time of the nonlinear sources in the backgrtound is controled by the constitutive relations.

These strategies have their roots in the dynamic processes in linearized nonstationary thermoelasticity, which have been explored in detail and the methods of analysis are well documented in text books and in the international journals of mechanics. Commonly the coupling of temperature and displacement fields is neglected in low frequency analysis of the vibrations of structures. That means a given driving agency by the imposed gradient of temperature acts on the time invariant dynamic system, and for temperature insensitive elastic stiffness, the system has constant parameters. Most importantly, the superposition principle is applicable and the solution is split into the quasistatic one (no inertia is taken into account) and the dynamic one. In such a case the dynamic boundary conditions can be made homogeneous and the dynamic solution might be suitably well documented by taking a low order modal expansion into account. Similar arguments apply to linearized thermoelastic waves. However, thermo-mechanical field coupling becomes important at shock fronts. It is the main content of the paper to transfer as many as possible of the nice and effective parts of the linear analysis to model the response of the plastically deforming structure. Since the virgin state of all structures is linear elastic we refer all the processes to a linear elastic background structure of same geometry undergoing isothermal deformations however subjected to the given loads and in addition forced by distributions of sources of eigen-stresses (distortions). We keep the geometric relations linearized but refer to some problems of engineering importance (beams and plates) where also some effects of geometric nonlinearity are approximately included in the analysis along the same lines of internal sources. The ingredient to the nonlinear analysis is the static or the dynamic influence function of the background structure derived for nonhomogeneous initial conditions. The main tool of analysis of structural vibrations is the dynamic form of a generalized Maysel's formula (well-known from linear thermo-elastic statics). A derivation of that formula will be given by taking into account all kinds of noncompatible strains in the background structure. The solution based on this analysis is built-in into a time incremental numerical solution technique. Thereby, the updating in space and time of all the internal sources, e.g., by considering the constitutive equations, is done in a subroutine manner. Within the time increment, the superposition principle holds true. A strong thermal shock applied to a ductile beam serves as an illustrative example. Even material damage in the plastic zones can be taken into account, e.g., by the model of plastic growth of voids. In open finite element codes those internal sources could be considered in a straight forward manner. In case of viscoplastic waves local sources of plastic strain act in the background in the case of uni-axial D'Alembert waves. However, in the case of spherical waves, nonlocal sources must be considered in the background, i.e. to each plastic source in a thin spherical cell an outward pointing tail is connected. Superposition of the elastic waves thus produced render all kinds of phenomena like plastic shock fronts, unloading waves etc. An outlook is given to extend the solution to three-dimensional problems, say of waves propagating in an elastic-viscoplastic (layered) half-space by means of the linear method of generalized ray integrals in the background. Finally, the stable numerical algorithm associated to these formulations should be mentioned, which is partly due to the load effect by integration, with some smoothing effect on any errors in the location and intensity of the updated sources in the background.

References

[1] H. Irschik and F. Ziegler: Dynamic Processes in Structural Thermo-Viscoplasticity. Applied Mechanics Reviews, AMR 48, no 6, 1995, 301- 316.

[2] C. Adam and F. Ziegler: Moderately large forced oblique vibrations of elastic-viscoplastic deteriorating slightly curved beams. Archive of Applied Mechanics 67 (1997), 375-392.

[3] C. Adam and F. Ziegler: Forced flexural vibrations of elastic-plastic composite beams with thick layers. Composites Part B 28B (1997) 201-213. (dedic. N.J. Hoff-90th birthday).

[4] C. Adam and F. Ziegler: Vibrations of layered beams under conditions of damaging and slipping interfaces. Int. J. Fracture 4466-GD The P.S. Theocaris volume (in press)

[5] W. Brunner, H. Irschik und F. Ziegler: Interaktionsprobleme im plastischen Bereich von Behälterschwingungen. Minisymposium "Fluid-Struktur Kopplung" ZAMM 78 (1998), Suppl. 1, S161 - S164 (Proc. GAMM-Tagung Regensburg 1997).

[6] H. Irschik, H. Holl and F. Ziegler: Spherical Elastic-Plastic Waves. Journal of Vibration and Control, 1, No. 3, 1995, 345-360.

[7] P. Fotiu and F. Ziegler: The Propagation of Spherical Waves in Rate-Sensitive Elastic-Plastic Materials. Int. Journal of Solids and Structures 33, 1996, No. 6, 811 - 833.

[8] H. Irschik and F. Ziegler: Elastic-plastic waves by superposition in the elastic background. Minisymposium MS 7 "Hyperbolic Equations with Source Terms" ZAMM 80 (2000), Suppl....(Proc. GAMM-Tagung Metz 1999) (submitted for publication).