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A 3D finite element approach to flexible multibody systems

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Abstract
The trend to weight-optimization of complex structures and machines leads to problems like resonance, noise or damage which are not predictable by analyzing the behavior of the single bodies only. In the present paper we combine existing techniques from multibody systems and from numerical analysis in order to contribute to the field of 3D deformable multibody dynamics. Two different approaches are used. The first method approximates the single bodies using a standard finite element modeling including only small deformations. The deformation is split into a rigid (rotation) and a flexible part. Constraints are introduced by integrals of the deformation of constrained surfaces. The resulting system of equations is transformed such that only a small system of equations compared to the elastic degrees of freedom has to be solved in every timestep. The formulation is capable to treat small nonlinear effects like geometric stiffening by introducing additional rotational degrees of freedom for every single body. The second method uses a Lagrangian description of the deformation including large deformation, displacements and rotations. The motion of the single bodies is constrained by contact, using a soft-contact model. The two methods are compared by means of a numerical simulation of a slider-crank mechanism. Numerical results are discussed with respect to efficiency and accuracy.

1 Introduction

The increasing computational power of nowadays computer enables to model a multibody system as 3D deformable body using a standard finite element discretization. The simulation of motion for a reasonable amount of simulated time can be performed within several hours of computational time. The main advantage of the finite element modeling is the possibility for the inclusion of effects like contact, geometric stiffening and nonlinear material behavior. However in practice, multibody systems are still mostly computed using simplifications to 1D or 2D bodies. In commercial software, the single bodies are either approximated as rigid bodies or with a few eigenmodes or elastic degrees of freedom.
For the appropriate modeling of 3D multibody system (MBS), some of the existing MBS-techniques are regarded. The motion of rigid body and deformable MBSs can be computed efficiently by a set of minimal coordinates, see e.g. Bremer and Pfeiffer [1]. Clearly, the deformation of the single bodies can not be computed accurately by means of an uncoupled computation of the rigid body MBS and a deformation analysis of the single bodies using the computed inertial forces of the MBS, see e.g. Gerstmayr [4]. For the computation of more complex flexible multibody systems a formulation with constraint conditions seems to be appropriate. The constraint formulation leads to simpler equations and is open to various kinds of nonlinearities and inequalities, however, the resulting discretized system of equations forms a set of differential algebraic equations (DAEs), see e.g. Eich-Soellner and Führer [2]. The application of numerical time-integration schemes generally leads to a reduction of the order of convergence (compared to ordinary differential equations) or even instability and therefore only special implicit Runge-Kutta or BDF formulas amoung other integration schemes are appropriate. The heaviness of the algebraic constraint condition in DAEs is generally described by the index of the system. Stabilization techniques, like the Gear-Gupta-Leimkuhler (GGL) stabilization method, reduce the index of the system, but lead to additional difficulties, like numerical drift or additional computational effort, see e.g. Simeon [11].
For the modeling of the deformation of a multibody system, different approaches exist. The floating frame of reference method uses a reference frame for the description of the deformation of the body while inertial forces lead to the nonlinear coupling, for details see Shabana [10]. The finite segment method splits the single bodies into small segments connected with springs and dampers. It therefore can be interpreted as a nonlinear finite element formulation using piecewise constant shape functions. Simo and Vu Quoc [12] introduced the large rotation vector formulation to efficiently describe the motion and large deformation of beam, plate and shell structures. The absolute nodal formulation uses finite elements and constraints without a reference frame. While it naturally leads to problems in the higher frequencies and even to instabilities, see also Shabana [10], stabilization techniques exist which add an artificial numerical damping, like the HHT method, see e.g. Hairer et al. [6, 7]. Alternatively, Gonzalez and Simo [5] derived stable energy-momentum methods for Hamiltonian systems and it has been extended to contact and impact problems by Demkowicz and Bajer [3].
The finite element method has been used for the modelling of multibody dynamics problems with contact, see e.g. Orden and Goicolea [8] with a numerical example of an impact of a double pendulum or see Simeon [11] for the numerical example of a two-dimensional slider-crank mechanism.
The possibility of constructing general geometries (e.g. a crankshaft) cheaply and in large numbers increases the demand for 3D finite element multibody methods. Standard approaches for 3D multibody systems which use the component mode synthesis in order to reduce the number of degrees of freedom show a lack in the modelling of contact or material nonlinearities. In the present paper, we first use a 3D finite element approach, where we assume large displacements but small deformations of the single bodies for simplicity and efficiency. The underlying rigid body motion is described by an overall displacement and rotation vector which is calculated by the linear regression of the displacement of the

single bodies. The formulation is a combination of the above mentioned floating frame of reference formulation and the absolute nodal formulation. The subsequently defined rotation matrix is similar to a constant approximation of the large rotation vector. A set of linear constraints is introduced for arbitrary parts of the surfaces of the bodies. Constraints are described in integral terms, where the displacement of one surface (for ground joints) or the distance of two surfaces (for body joints) is weighted and integrated over the whole surfaces. Revolute, translational or sherical joints may be easily introduced in this manner. At every stage of the time-integration method, the solution is projected into the constraint-space. An efficient solution strategy is used which takes advantage of the small number of constraint equations but large deformable degrees of freedom. At the current state, the formulation only regards small deformations and linear material, however, it is open to nonlinear effects like stiffening effects of thin bodies or nonlinear material effects in the range of small strains. The results of this method are compared to a special implementation of the 3D finite element method including large deformations and a soft-contact model. The material (Lagrangian) description method is used, for details see e.g. Zienkiewicz [14]. For details on methods in contact problems see Wriggers [13]. The formulation of the penalty-like contact model leads to a nonlinear system of equations in every timestep which is solved by the Newton method. No additional loop is necessary in order to fulfill the contact conditions. An alternative, efficient contact solver based on domain decomposition techniques was presented in Schöberl [9]. As a numerical example, the motion of a 3 dimensional slider-crank mechanism is studied.
All computations have been implemented and tested within the 3D finite element package NGSolve, the meshes have been generated by the mesh generator Netgen. This software is available for free for non-commercial purposes, see the WWW-link *www.sfb013.uni-linz.ac.at/∼joachim/netgen*.

1.1 3D finite element constraint formulation

In this section, we describe a finite element formulation which incorporates large rotations, but only small deformations. Its advantage is that the stiffness matrix is assembled once and for all. It requires just one factorization during the initialization phase.

The equations of motion are derived from Hamilton's principle, which states that the variation of the Lagrangian $L = T - V$ vanishes, that is $\delta \int_{t1}^{t2} L dt = 0$. The kinetic energy is expressed by $T = 1/2 \int \rho |\dot{u}|^2 \, dx$ and V describes the potential energy of the mechanical system. Integration by parts in time leads to the equation

$$\rho \ddot{u} = -\nabla_u V(u).$$

The derivative $\nabla_u V(u)$ is understood in week sense, which is defined as action on a test function v, namely $\langle \nabla_u V(u), v \rangle := \lim_{t \to 0} [V(u + tv) - V(u)]$. For hyperelastic materials, the potential energy is given as

$$V(u) = \int_\Omega W(C(u)) \, dx - \int_\Omega f \cdot u \, dx,$$

where $W(.)$ is the hyperelastic energy functional, and $C(u) = (I + \nabla u)^T (I + \nabla u)$ is the Cauchy Green strain tensor. The body-force f is defined over the reference domain Ω. For a linear material, the hyperelastic energy functional is the quadratic form $W(C) = \mu \, |C - I|^2 + \lambda/2 \, \{\text{trace}(C - I)\}^2$ with the Lamé coefficients μ and λ.

A geometrically linear formulation neglects second order terms in ∇u, i.e. $C \approx I + \nabla u + (\nabla u)^T$. This simplification cannot describe large rotations, since $\tilde{C} \neq I$ for rigid body displacements u. We decompose

$$u = u_0 + \tilde{u},$$

where u_0 represents large rigid body displacements and $\tilde{u}$ is the *small* deformation. We approximate C by neglecting second order terms in $\tilde{u}$:

$$C \approx \widetilde{C} := (I + \nabla u_0)^T (I + \nabla u_0) + \nabla u_0^T \nabla \tilde{u} + \nabla \tilde{u}^T \nabla u_0.$$

The decomposition of u is a priori not unique. Even the number of unknowns for the large deformation part can be modified. One possibility is to choose u_0 in a finite element space on a much coarser mesh. A minimal requirement is that rigid body deformations go into u_0. To define the method we have to come up with an explicit mapping $u \to u_0$. We suggest the following procedure. First, compute the L_2-best-approximation of u by an affine-linear function in each body. Then, fix three points to define a translation u_t and a rotation matrix R. The rigid body part is $u_0(x) = u_t + Rx - x$.

The kinematic constraints are introduced by Lagrange parameters. The set of constraints forms a small number of equations which is independent of the underlying mesh. Possible constraints are prescribed mean-values of the deformation on a part of the surface, mean-values of the rotation, and the coupling of mean-values of two different bodies. Gerneralized, we take

$$B(u) = 0$$

to be a finite number of constraint conditions. First, we approximate the constraints by a penalty term and define the new potential energy by

$$V_{total}(u) = V(u) + 1/(2\varepsilon)\|B(u)\|^2.$$

It's derivative is

$$\nabla_u V(u) + 1/(\varepsilon) B(u)^T \nabla_u B(u),$$

and the equations of motion for the multi-body system follow to

$$\rho \ddot{u} + \nabla_u V(u) + 1/\varepsilon B(u)^T \nabla_u B(u) = 0.$$

Introducing the new variable $\lambda := 1/\varepsilon B(u)$ leads to the mixed system

$$\begin{array}{rcrcl} \rho\ddot{u} + \nabla_u V(u) & + & B'(u)^T\lambda & = & 0, \\ B(u) & - & \varepsilon\lambda & = & 0. \end{array}$$

Now, we can pass to the limit $\varepsilon = 0$. Finally, we recover the variable λ as constraint force.

The finite element discretization leads to ordinary differential equations. Explorating the special structure of the energy $V(u)$ results in

$$\begin{array}{rcrcl} M\ddot{u} + R(u)^T A R(u) u & + & B'(u)^T\lambda & = & f \\ B(u) & & & = & 0. \end{array}$$

Due to the small number of constraints, the function $B(u)$ is rewritten as $B(u) = B_1(B_2 u)$, where B_2 is a block-diagonal matrix with respect to x, y, and z components. Thus, B_2 commutes with the rotation matrix R. The mass matrix commutes with R as well. Standard time-stepping methods lead to linear equations of the form

$$\begin{array}{rcrcl} (M + \tau^2 A)Ra & + & B_2^T R B_1'(u)^T \lambda & = & Rf + d, \\ B_1' R^T B_2 R a & & & = & e, \end{array}$$

where a is the acceleration and d and e are terms which only depend on known quantities of the last time-step and are given by the specific time-stepping method.

These systems can be solved efficiently: Eliminating Ra from the first line leads to the Schur-complement equation

$$B_1' R^T B_2 (M + \tau^2 A)^{-1} B_2^T R B_1'^T \lambda = B_1' R^T B_2 (M + \tau^2 A)^{-1} (Rf + d) - c.$$

For a finite number of different time steps τ, the matrices $B_2(M + \tau^2 A)^{-1} B_2^T$ can be computed in advance. Thus, the most expensive part of the computation is moved out of the time integration loop. It remains to multiply twice with the matrix $(M + \tau^2 A)^{-1}$, which is also factorized in advance.

1.2 3D finite element contact formulation

An alternative approach to multibody systems is based on the fully nonlinear strain tensor and body-body contact conditions. The contact is described by a penalty formulation. The term V_c measuring the penetration is added to the potential energy $V(u)$. In the initial step one computes the function $d : \Omega \to \mathbf{R}^+$ on each body, which is an approximative measure of the distance to the boundary. Outside we set $d = 0$. Then, the penalty term is defined as

$$V_c(u) := \int_{\partial\Omega} \gamma \hat{d}(x + u(x))^2 \, ds,$$

where γ is a proper penalty parameter, which is chosen proportional to stiffness divided by mesh-size. The integral is taken over the reference domain, while $\hat{d}(.)$ must be evaluated on the deformed domain and is defined as

$$\hat{d}(\hat{x}) := \max\{d(x) : x + u(x) = \hat{x}\}.$$

Here, geometric search trees must be used to keep the computational complexity inside a reasonable range.

1.3 Numerical example: 3D slider-crank mechanism

The aim of the numerical example is to experience the differences of the two methods. As a numerical example, a 3D slider-crank mechanism is treated, for a sketch of the geometry, see Fig. 1. Only to have an idea of the dimensions of the mechanism we specify the displacement of the slider which is in a range of 40mm. The driving element is accelerated for the first 0.1 seconds, one cycle of the mechanism lasts about 0.8 seconds. While the radius of the shaft is 5mm, we introduce a quite large bearing clearance of 0.1mm in the contact model, only to point up the differences of the two models. While the overall behavior measured by the large displacements of the bodies or the cycle time of the system are almost the same for both models, the deformation shows larger differences. As an example, we compare the deflection of the crankshaft defined by the distance of the midpoints of the two outmost cylinders to the midpoint of the eccentric shaft for both methods. In the case of the proposed constraint formulation (subsequently called method 1), we take the 2-stage RadauIIA method which is stable and accurate also for larger timesteps (≈0.05 seconds). Fig. 2 depicts the deflection of the crankshaft using method 1. The deflection mainly results due to inertial forces forces of the slider and the connecting rod. The harmonic oscillations reflect the idealized modelling due to constraints, the peaks at 0.1 seconds results due to the abrupt turn-off of the load. In case of the contact formulation (subsequently called method 2) we apply the Newmark-scheme, which leads to small timestep-sizes (10^{-4} - 10^{-3}) but smaller computational

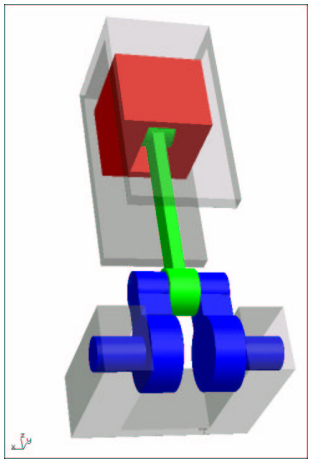

Figure 1: Geometry of the slider-crank mechanism.

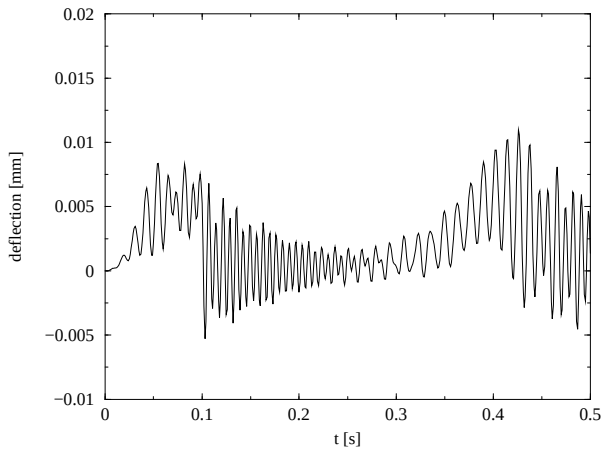

Figure 2: Deformation of the crankshaft with constraint formulation.

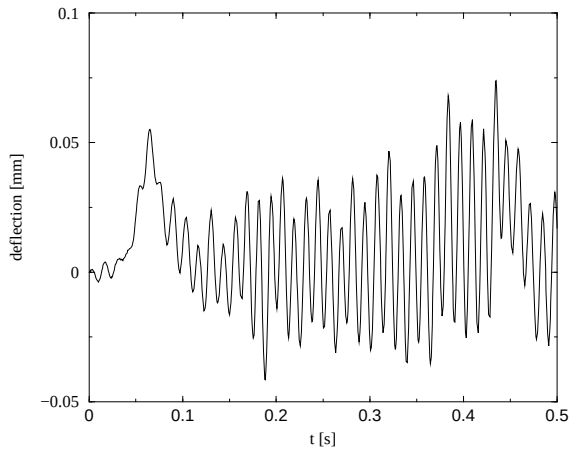

Figure 3: Deformation of the crankshaft with contact formulation.

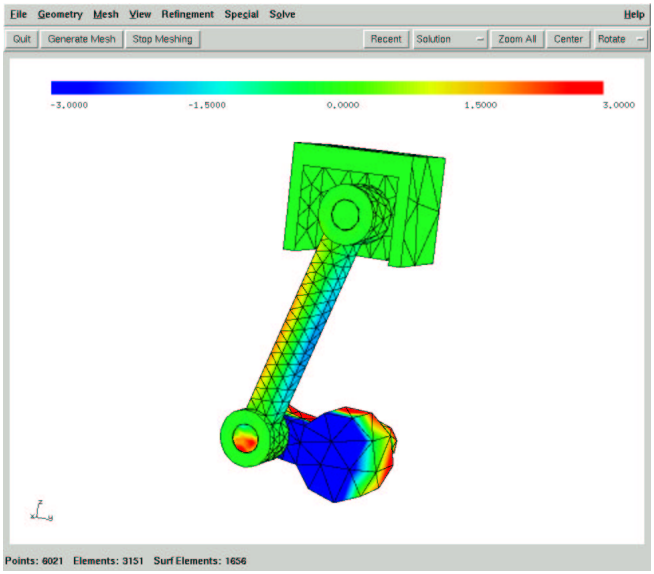

Figure 4: 3D view of the mechanism, contour-plot of stress component σ_{zz}.

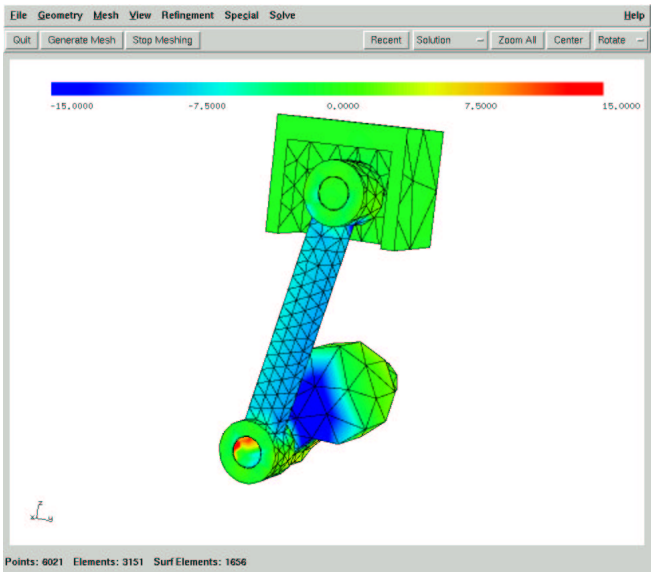

Figure 5: 3D view of the mechanism, contour-plot of stress component σ_{zz}.

effort in the single timesteps which have to be kept small anyway in order to resolve the impacts of the contacts. Fig. 3 shows the output for the deflection of the crankshaft using method 2. Compared to Fig. 2, a larger deflection results due to impacts in the bearings. The study of the reduction of the bearing clearance and a convergence to the constraint formulation is currently under investigation. Fig. 4 and Fig. 5 show detailed plots of the Cauchy stress components σ_{zz} at two different instances in time for method 1 using second order elements (#elements=3151, #nodes=6021). As expected, method 1 is computationally more efficient because only a small system of equations is solved in every timestep, while in method 2 the Newton method is applied for the whole system in every timestep. While method 1 needs several minutes for the computation of one rotation of the crankshaft, method 2 took about 10 hours on a SGI 500 MHz R14000 processor for a rather coarse approximation of the geometry.

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