Numerical Methods in Continuum Mechanics II

Tutorial 1

October 15, 2007

In some of the exercises throughout the tutorial you will have to implement code in OCTAVE or MATLAB. Therefore you can work on one of the computers in Room 519 in Kopfgebäude 5th floor. Ask the system administrator Oliver Koch for an account if you don't have one (email: `koch@numa.uni-linz.ac.at`).

Probably you prefer to implement on your own computer. If you do not have OCTAVE or MATLAB installed, you may download and install OCTAVE for free. Please, follow the link `http://www.gnu.org/software/octave/octave.html`.

1. Let $\Omega \subset \mathbb{R}^2$ be a polygonal domain with $n \geq 3$ vertices

   $$(x_1, y_1),\ (x_2, y_2),\ \ldots,\ (x_n, y_n),\ (x_{n+1}, y_{n+1}) = (x_1, y_1)\,.$$

   Find a formula (which is easy to implement) for calculating its area $|\Omega|$.

2. Let OCTAVE or MATLAB plot an arbitrary polygonal domain, where $n \geq 5$, and calculate its area via the formula of Example 1. Do the same for a nice triangle ($n = 3$) and verify the output by calculating its area by hand.

3. The Green–St. Venant strain tensor $E(x) := \frac{1}{2}\big(\mathrm{D}\varphi^T(x)\,\mathrm{D}\varphi(x) - I\big)$ displays the relative variation of lengths in the sense of

   $$\|\varphi(x + \mathrm{d}x) - \varphi(x)\|^2 - \|\mathrm{d}x\|^2 = \langle E(x)\,\mathrm{d}x\,,\ \mathrm{d}x\rangle + \mathbf{o}(\|\mathrm{d}x\|^2)\,.$$

   It can be shown, that E vanishes for all $x \in \Omega$ if and only if the deformation satisfies $\varphi(x) = Q\,x + b$ with $b \in \mathbb{R}^3$ and $Q \in \mathbb{R}^{3\times 3}$ orthogonal with $\det Q = 1$. These deformations are called the rigid body motions, and can be seen as the kernel of E as a function in φ. By writing E in terms of the displacement $u(x) = \varphi(x) - x$ we obtain

   $$E(x) = \frac{1}{2}\left(\mathrm{D}u(x)^T + \mathrm{D}u(x) + \mathrm{D}u(x)^T\,\mathrm{D}u(x)\right)\,.$$

   The assumption, that $\|\mathrm{D}u(x)\|$ is small for all x, motivates the definition of the linearized strain tensor $\varepsilon(x) = \frac{1}{2}\big(\mathrm{D}u(x) + \mathrm{D}u^T(x)\big)$. Proof, that the kernel of ε amongst $u \in C^2(\Omega)$ is given by

   $$\ker\varepsilon = \{u : \Omega \to \mathbb{R}^3 \mid u(x) = a \times x + b \ \text{ with } a,\, b \in \mathbb{R}^3\}\,.$$

   Hint: Use the identity

   $$\frac{\partial\,\varepsilon_{ij}}{\partial\,x_k} + \frac{\partial\,\varepsilon_{ki}}{\partial\,x_j} - \frac{\partial\,\varepsilon_{jk}}{\partial\,x_i} = \frac{\partial^2\,u_i}{\partial\,x_j\,\partial\,x_k} \qquad \forall\, i,\ j,\ k \in \{1, 2, 3\}\,.$$