4. The faces of this tetraeder are named $F_x$ (to which the $x$-axis is normal), $F_y$ (to which the $y$-axis is normal), $F_z$ (to which the $z$-axis is normal), and $F$ (to which $\vec{n} = (n_x, n_y, n_z)^T$ with $\|\vec{n}\| = 1$ is normal). Show, that the areas of the faces satisfy $|F_i| = n_i |F|$ for all $i \in \{x, y, z\}$.

5. Last part of Cauchy’s theorem: Let $\Omega$ be a bounded domain, $f : \Omega \to \mathbb{R}^3$ a given bodyforce, and $t(x, n)$ be Cauchy’s stress vector acting at the material point $x$ with respect to the cut surface regarding the normal direction $n$. Let $t$ be such, that for all subsets $A \subset \Omega$ there holds
\[
\int_A x \times f(x) \, dx + \int_{\partial A} x \times t(x, n) \, ds = 0.
\]
Assume that there exists a mapping $\sigma : \Omega \to \mathbb{R}^{3 \times 3}$, such that $t(x, n) = \sigma(x)n$ and $-\text{div} \, \sigma(x) = f(x)$ hold for all $x \in \Omega$. Show, that $\sigma(x) = \sigma(x)^T$ for all $x \in \Omega$.

6. Let $A, B$ be square matrices. The Frobenius scalar product and norm are defined
\[
\langle A, B \rangle_F := \sum_{ij} a_{ij}b_{ij}, \quad \|A\|_F := \langle A, A \rangle_F^{1/2}.
\]
Moreover, we define the trace of $A$ by $\text{tr} \, A := \langle A, I \rangle_F$ and the deviator of $A$ by $\text{dev} \, A := A - \frac{\text{tr} \, A}{\text{tr} \, I} I$, where $I$ denotes the identity matrix. Verify both identities
\[
\text{tr}(QAQ^T) = \text{tr} \, A, \quad \|\text{dev}(QAQ^T)\|_F = \|\text{dev} \, A\|_F,
\]
where $Q$ is an orthogonal matrix, i. e., $Q^TQ = I$.

7. Let $\sigma \in \mathbb{R}^{3 \times 3}$ be symmetric with eigenvalues $\sigma_1, \sigma_2, \sigma_3 \in \mathbb{R}$. By using the definitions and results(!) of Example 6, show that there holds
\[
\sqrt{\frac{3}{2}} \|\text{dev} \, \sigma\|_F = \sqrt{\frac{(\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 + (\sigma_1 - \sigma_2)^2}{2}}.
\]