Numerical Methods in Continuum Mechanics II

Tutorial 2

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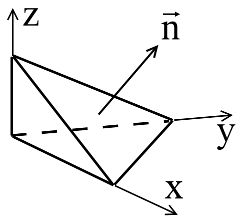

4. The faces of this tetraeder are named F_x (to which the x-axis is normal), F_y (to which the y-axis is normal), F_z (to which the z-axis is normal), and F (to which $\vec{n} = (n_x, n_y, n_z)^T$ with $\|\vec{n}\| = 1$ is normal). Show, that the areas of the faces satisfy $|F_i| = n_i\,|F|$ for all $i \in \{x, y, z\}$.

5. *Last part of Cauchy's theorem:* Let Ω be a bounded domain, $f : \Omega \to \mathbb{R}^3$ a given bodyforce, and $t(x, n)$ be Cauchy's stress vector acting at the material point x with respect to the cut surface regarding the normal direction n. Let t be such, that for all subsets $A \subset \Omega$ there holds

$$\int_A x \times f(x)\,\mathrm{d}x + \int_{\partial A} x \times t(x, n)\,\mathrm{d}s = 0\,.$$

Assume that there exists a mapping $\sigma : \Omega \to \mathbb{R}^{3\times 3}$, such that $t(x, n) = \sigma(x)n$ and $-\operatorname{div}\sigma(x) = f(x)$ hold for all $x \in \Omega$. Show, that $\sigma(x) = \sigma(x)^T$ for all $x \in \Omega$.

6. Let A, B be square matrices. The Frobenius scalar product and norm are defined

$$\langle A\,,\, B\rangle_F := \sum_{ij} a_{ij} b_{ij}\,, \qquad \|A\|_F := \langle A\,,\, A\rangle_F^{1/2}\,.$$

Moreover, we define the *trace* of A by $\operatorname{tr} A := \langle A\,,\, I\rangle_F$ and the *deviator* of A by $\operatorname{dev} A := A - \frac{\operatorname{tr} A}{\operatorname{tr} I} I$, where I denotes the identity matrix. Verify both identities

$$\operatorname{tr}(QA\,Q^T) = \operatorname{tr} A\,, \qquad \|\operatorname{dev}(QA\,Q^T)\|_F = \|\operatorname{dev} A\|_F\,,$$

where Q is an orthogonal matrix, i. e., $Q^T Q = I$.

7. Let $\sigma \in \mathbb{R}^{3\times 3}$ be symmetric with eigenvalues $\sigma_1,\, \sigma_2,\, \sigma_3 \in \mathbb{R}$. By using the definitions and results(!) of Example 6, show that there holds

$$\sqrt{\frac{3}{2}}\,\|\operatorname{dev}\sigma\|_F = \sqrt{\frac{(\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 + (\sigma_1 - \sigma_2)^2}{2}}\,.$$