Numerical Methods in Continuum Mechanics II

Tutorial 4

November 15, 2007

11. *Mesh refinement – round corners:* Download and extract the file `wrench.zip` from `www.sfb013.uni-linz.ac.at/~peter/`. This will create the directory `wrench` with the files `readme.txt`, `elements.mat`, `nodes.mat`, `surface.mat`, and `spline2.mat`. Take a look at `readme.txt`. Then, extend the Matlab function of Example 9 by the facility of creating round corners during the mesh refinement. Therefore, follow the instructions in Figure 1.

12. Show, that after infinitely many refinement steps, the strategy outlined in Figure 1 would produce the unique quadratic polynomial p which passes through `A` and `C`, and whose slope equals $\vec{\mathtt{AB}}$ in `A`, and $\vec{\mathtt{BC}}$ in `C`.

13. Assume the vector representation of the stress σ and the strain ε by

$$\vec{\sigma} = \begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{pmatrix}, \quad \vec{\varepsilon} = \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\,\varepsilon_{12} \\ 2\,\varepsilon_{23} \\ 2\,\varepsilon_{31} \end{pmatrix}.$$

Show, that $\vec{\sigma} = C\,\vec{\varepsilon}$ represents Hooke's law $\sigma = 2\mu\varepsilon + \lambda\,\mathrm{tr}\,\varepsilon I$ (where $\lambda > 0$ and $\mu > 0$), if

$$C = \begin{pmatrix} \lambda+2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda+2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda+2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{pmatrix}.$$

Show, that C is positive definite, and calculate its condition number $\kappa = \frac{\lambda_{\max}}{\lambda_{\min}}$, and its inverse.

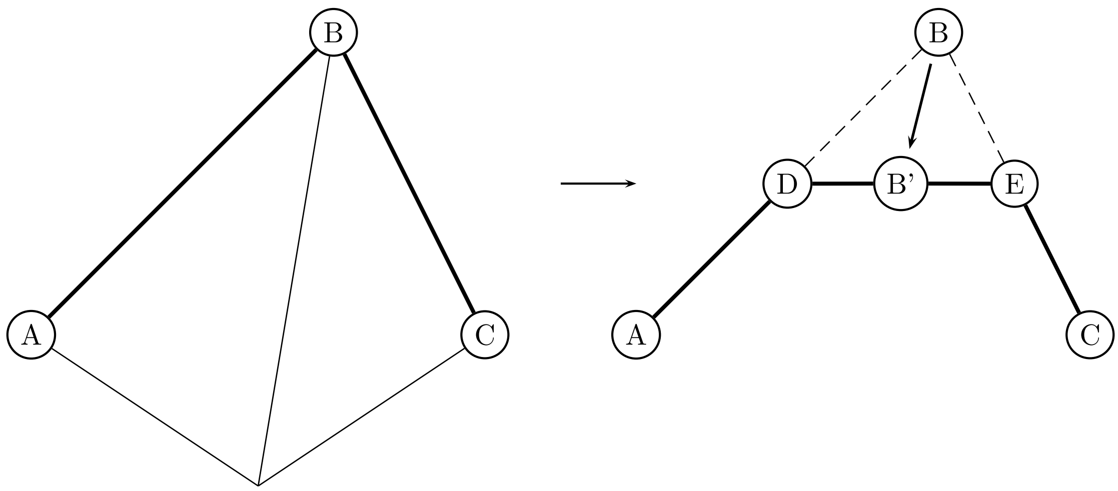

Figure 1: In the left picture you can see two segments $\vec{AB}$ and $\vec{BC}$ which we assume to be edges of two different triangles. If one does usual mesh refinement, the segments would be bisected (see D and E in the right picture) as the related triangles are split into quarters – nothing more. If one would like to obtain a round courner between A and C, this can be obtained by additionally shifting the vertex B to a new position B' in the middle between D and E. Recursively, in the next refinement step one would apply the same strategy seperately for both double segments ADB' and $B'EC$, as was done for the double segment ABC before.