# Numerical Methods in Continuum Mechanics II

Tutorial 8

December 13, 2007

25. Implement Example 21 of Tutorial 7 in Matlab. Therefore, download and extract `square.zip` from the homepage. Here you will find the same files as we already had before, but also two newly added files: `dirichlet.mat` and `neumann.mat`. Take a look at `readme.txt` how these files are used. Modify your Matlab function of Example 9 and 11 (refinement), such that the files `dirichlet.mat` and `neumann.mat` are taken into respect. Then, solve Example 21 by implementing a Matlab function

    `[K,u,f]=laplace(elements,nodes,surface,dirichlet,neumann)`,

    which returns the stiffness matrix $K_h$, the right handside $\underline{f}_h$ and the solution $\underline{u}_h$. Notice and use in your implementation, that on a triangle with vertices $(x_i, y_i)$, $(x_j, y_j)$, and $(x_k, y_k)$, the gradients of the basis functions can be calculated by

    $$\begin{pmatrix} \nabla\varphi_i \\ \nabla\varphi_j \\ \nabla\varphi_k \end{pmatrix} = F^{-1} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, \text{ with } F := \begin{pmatrix} x_i & x_j & x_k \\ y_i & y_j & y_k \\ 1 & 1 & 1 \end{pmatrix}.$$

    Moreover, the area of such triangle can be calculated by $\det(F)/2$ (cf. Example 1 of Tutorial 1). Use the midpoint rule for the numerical integration of the Neumann boundary conditions. Plot the solution $\underline{u}_h$ with `trimesh`.

26. Solve Example 25.

27. Implement an elasticity solver in Matlab via the function

    `[K,u,f]=elasticity(elements,nodes,surface,dirichlet,neumann)`.

    Apply it to the wrench mesh, which you should download and extract once again from the homepage (`wrench.zip`), since the files `dirichlet.mat` and `neumann.mat` have been added. See Figure 1 for a short description of the problem. Since this is a 2D mesh, the displacement $u$ has two components. Thus, use the decomposition $u_h(x) = \sum_{i=1}^{n} \sum_{j=1}^{2} u_{j,i}\, \varphi_i(x)\, \underline{e}_j$, where $\underline{e}_j$ denotes the $j$th unit vector, and $\varphi_i(x)$ are linear triangle basis functions just like in Example 25. In order to apply the results of 3D-elasticity theory correctly to this 2D-mesh and the 2D-version of the displacement $u$, we assume the strain $\varepsilon$ to satisfy $\varepsilon_{13} = \varepsilon_{23} = \varepsilon_{33} = 0$ (called: *plain strain model*), hence use the representation $\varepsilon = (\varepsilon_{11}, \varepsilon_{22}, 2\,\varepsilon_{12})^T$. Calculate the $L_2$-norm of the stress on each triangle and plot it by using the command `trisurf`. Visualize the deformation of the wrench by adding the displacement $u_h$ to `nodes`.

28. Solve Example 27.

**Note:** The deadline for all of these examples is the 10th of January. Nevertheless, we meet at the next tutorial on the 13th of December and discuss the examples in detail. In particular, we will discuss questions concerning the implementation, so try to get as far as possible with solving the examples.

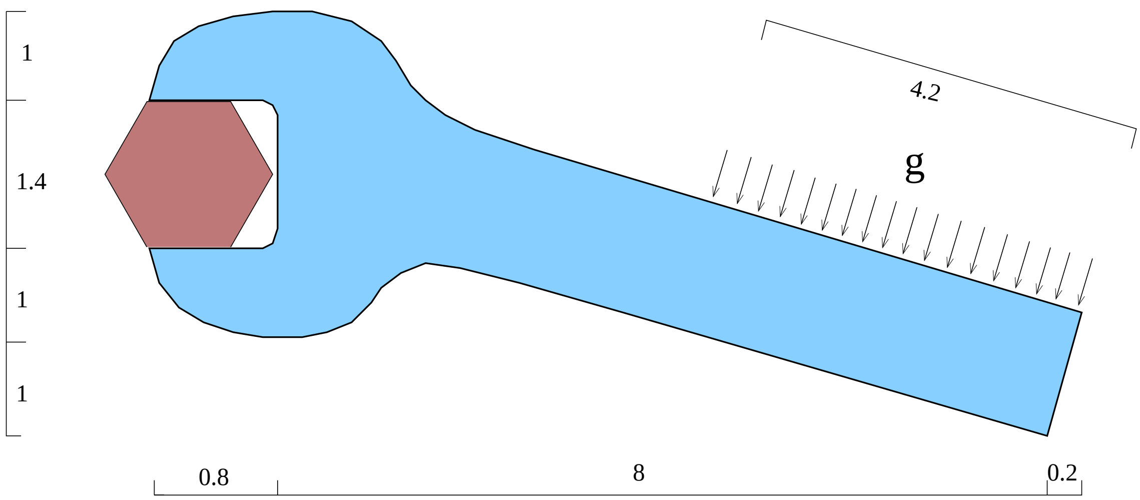


Figure 1: This wrench sticks on a screw (homogeneous Dirichlet condition) and is pressed down at the handheld with a surface force $g$ in normal direction with intensity $|g| = 6e4$ (Newton per length unit). The material constants read $\lambda = 1.15e08$ and $\mu = 7.7e07$.