Numerical Methods in Continuum Mechanics II

Tutorial 9

January 17, 2008

29. Let V denote a function space and $V_D \subset V$ its subset (hyper plane) of functions which satisfy the Dirichlet condition $v = u_D$ on Γ_D. Let $a(\cdot,\cdot)$ be a bilinear form, $\langle F \,,\, \cdot\rangle$ a linear form, $\psi : V_D \to \mathbb{R}$ a convex function, and $J(v) := \frac{1}{2}a(v,v) + \psi(v) - \langle F \,,\, v\rangle$. Show that $u \in V_D$ solves the variational inequality
$$a(u, u-v) + \psi(u) - \psi(v) \leq \langle F \,,\, u-v\rangle \quad \forall v \in V_D\,,$$
if and only if it satisfies
$$J(u) = \inf_{v\in V_D} J(v)\,.$$

30. When modeling elastoplasticity with the kinematic hardening scheme (cf. lectures) the bilinearform of the variational inequality reads
$$a((u,p),(v,q)) = \int_\Omega \mathbb{C}\,(\varepsilon(u) - p) : (\varepsilon(v) - q) + \mathbb{H}\,p : q \,\mathrm{d}x\,.$$
Here, $\mathbb{C}$ is the elastic stiffness tensor (Hooke's law), and $\mathbb{H}$ is a positive definite tensor of the same size and dimension as $\mathbb{C}$. Show that there exists $\alpha > 0$ such that $a((v,q),(v,q)) \geq \alpha\left(\|v\|^2_{H^1} + \|q\|^2_{L_2}\right)$ holds for all $(v,q) \in \left[H^1_0(\Omega)\right]^3 \times [L_2(\Omega)]^{3\times 3}_{\mathrm{sym}}$.

31. Let $A \in \mathbb{R}^{3\times 3}$ with $\operatorname{tr} A = 0$ and $b \in \mathbb{R}$ with $b > 0$. Find Q with $\operatorname{tr} Q = 0$ such that
$$J(Q) := \frac{1}{2}Q : Q - A : Q + b\|Q\|_F$$
attains its minimum.