Numerical Methods in Continuum Mechanics II

Tutorial 10

January 31, 2008

32. Let $A \in \mathbb{R}^{3\times 3}$ and ξ, $b \in \mathbb{R}$ with $b > 0$ and $\xi > 0$. Find Q with $\operatorname{tr} Q = 0$ such that

$$J(Q) := \frac{\xi}{2} Q : Q - A : Q + b\|Q\|_F$$

attains its minimum. Hint: $\operatorname{dev} A$ is trace free.

33. Let t_k with $k \in \mathbb{N}$ denote the k-th time step, $V_D := \{v \in \left[H^1(\Omega)\right]^3 \mid v = u_D \text{ on } \Gamma_D\}$, and $S := \{q \in [L_2(\Omega)]^{3\times 3}_{\mathrm{sym}} \mid \operatorname{tr} q = 0\}$. Write down the minimization problem $J_k(u_k, p_k) \to \min$ in detail for elastoplasticity with kinematic hardening (with the elastic stiffness tensor $\mathbb{C}$ for which there holds $\mathbb{C}\, x = 2\mu x + \lambda \operatorname{tr} x I$ and the hardening tensor $\mathbb{H}$ which we choose to be such that $\mathbb{H}\, x = hx$ with $h \in (0, +\infty)$, see the lectures). Show that, for fixed $\bar{u}_k \in V_D$, the minimizer $p_k \in S$, for which there holds $J_k(\bar{u}_k, p_k) \leq J_k(\bar{u}_k, q)$ for all $q \in S$, can be found via the minimization problem in Example 32. What are ξ, A and b then, and what does the minimizer p_k then look like in dependency of $\bar{u}_k$?

34. Let t_k with $k \in \mathbb{N}$ denote the k-th time step, $V_D := \{v \in \left[H^1(\Omega)\right]^3 \mid v = u_D \text{ on } \Gamma_D\}$, and $S := \{q \in [L_2(\Omega)]^{3\times 3}_{\mathrm{sym}} \mid \operatorname{tr} q = 0\}$. Show that, for fixed $\bar{p}_k \in S$, the minimizer $u_k \in V_D$, for which there holds $J_k(u_k, \bar{p}_k) \leq J_k(v, \bar{p}_k)$ for all $v \in V_D$, can be found via a linear system.